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273. 卵形線ノ一定理

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(定理) 對点ヲ擬似二重法線ヲ有セバ楕円ナリ。

(証明) $\gamma, \bar{\gamma}$ ハ對点トシ (Blaschke, IIヲ用ユ)

$$(1) \quad (\gamma' \gamma'') = 1, \quad K\gamma'(s) + \gamma'''(s) = 0$$

トオキ得, s ハ Affinbogen, K ハ Affinkrümmung
ナル。假定ニヨリ

$$(2) \quad \bar{\gamma} - \gamma = \alpha \cdot \gamma'' = -\alpha \bar{\gamma}''.$$

$\bar{\gamma}, \gamma$ = 於ケル Affinevoluten $\bar{\gamma}$ ハ一致スル故ニ

$$(3) \quad \bar{\gamma} = \gamma + \frac{1}{K} \gamma'' = \bar{\gamma} = \bar{\gamma} + \frac{1}{K} \bar{\gamma}''$$

且ツ

$$(4) \quad \bar{\gamma}' = \left(\frac{1}{K}\right)' \gamma'' = \bar{\gamma}' \frac{d\bar{\gamma}}{ds} = \left(\frac{1}{K}\right)' \bar{\gamma}'' \frac{d\bar{\gamma}}{ds}$$

又 (2), (1) ヨリ

$$(5) \quad \frac{d\bar{\gamma}}{ds} - \gamma' = \alpha \gamma''' + \alpha' \gamma'' = -\alpha K \gamma' + \alpha' \gamma''$$

而シテ $\gamma' \parallel \frac{d\bar{\gamma}}{ds}$ ナル故 $\alpha' = 0, \quad \alpha = \text{const} = \bar{\alpha}$ ナリ。

(5) ヨリ

$$(6) \quad \begin{cases} (\alpha K - 1) \gamma' = -\frac{d\bar{\gamma}}{ds} = -\bar{\gamma}' \frac{d\bar{\gamma}}{ds}, \\ \alpha = \bar{\alpha} = \text{const.} \end{cases}$$

(1), (4) ヨリ

$$(\alpha K - 1) \left(\frac{1}{K}\right)' = -\left(\frac{1}{K}\right)' \bar{\gamma}'^2$$

(6) ヨリ (2) ヲ用ヒテ

$$\begin{aligned} \alpha K' \varphi' + (\alpha K - 1) \varphi'' &= -\bar{\varphi}'' \bar{S}'^2 - \bar{\varphi}' \bar{S}'' \\ &= \varphi'' \bar{S}'^2 + \frac{\bar{S}''}{\bar{S}'} (\alpha K - 1) \varphi' \end{aligned}$$

故=

$$\begin{aligned} \frac{d \log(\alpha K - 1)}{dS} &= \frac{d \log \bar{S}'}{dS}, \quad \alpha K - 1 = \bar{S}'^2 \\ \alpha K - 1 &= c \bar{S}', \quad \bar{S}' = c \quad (c = \text{const}) \end{aligned}$$

又關ナル條件トシテ

$$c = \frac{d\bar{S}}{dS} = \frac{dS}{d\bar{S}} = \frac{1}{c}$$

$$\therefore \bar{S}' = c = 1, \quad \alpha K = 2 = \alpha \bar{K}$$

$$\therefore K = \bar{K} = \frac{2}{\alpha} = \text{const.} \quad \text{即チ証明完了。}$$